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INTUITIONISTIC FUZZY LOGIC IN BUSINESS EDUCATION

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Introduction

The purpose of this note is to introduce Intuitionistic Fuzzy Logic to university academics as a readily accessible research tool with a variety of applications. There are waiting applications: please read Professor Atanassov's books for detailed guidance when you attempt to set one up – or contact him by email at the Bulgarian Academy of Sciences. IFS can at times be simple but not easy, and at other times they can seem easy but not simple!

The Creator of IFL

Krassimir Todorov Atanassov (Bulgarian: Красимир Тодоров Атанасов), who created Intuitionistic Fuzzy Logic, is a Bulgarian mathematician, and a Corresponding member of the Bulgarian Academy of Sciences[1]. He is best known for introducing the concepts of Generalized Nets and Intuitionistic Fuzzy Sets, which are extensions of the concepts of Petri Nets and Fuzzy Sets, respectively.

"Fuzziness" is not "vagueness"[2]: Intuitionistic Fuzzy Sets can be precise and accurate when applied in a variety of fields as indicated in the journal, *Notes on Intuitionistic Fuzzy Sets.*

Krassimir Atanassov graduated in mathematics in Sofia University, in 1978, and defended his *Doctor of Philosophy* thesis in 1986. He became *Doctor of Technical (Computer) Sciences* in 1997, with a thesis on Generalized nets, and three years later defended a second higher doctorate, *Doctor of Mathematical Sciences*, with a thesis in the other field of his scientific interest - intuitionistic fuzzy sets.

Since 1995 he has been working in the Centre of Biomedical Engineering at the Bulgarian Academy of Sciences, which in 2010 was merged into the Institute of Biophysics and

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* **These papers are for internal discussion within CESA on topics related to the CESA Mission.**

[1] From Wikipedia

[2] Russell, Bertrand. 1923. "Vagueness". *Australian Journal of Philosophy.* 1(1): 84-92.

Biomedical Engineering. In 1998, he became full professor and in 2012 was elected Corresponding member of the Bulgarian Academy of Sciences. In 2013, he was awarded the "Pythagoras" Award for considerable contribution to science (Technical Sciences), awarded by the Ministry of Education and Science of Bulgaria. In 2013, he was elected Fellow of the *International Fuzzy Systems Association*.

IFL

The terms "Intuitionistic Fuzzy Logic" and "Intuitionistic Fuzzy Set" were coined by Professor Atanassov with the idea of emphasizing a further liberalization of the notion of set membership by the introduction of a "measure" of *non-membership* in addition to the "measure" of *membership*, with the provision that the sum of the two measures be less than 1. This restriction expresses a kind of "consistency" of the measures.

As a gentle introduction to fuzzy sets in decision making under uncertain conditions, the interested reader is referred to Nishad *et al*[3]. It contains an illustrated example with four sets of data.

We grew up with Aristotelian set theory in classical logic where an object either belonged to a set or did not belong to it: its membership was unity or zero, true (T) or false (F). While it is a caricature of fuzzy logics, if we define the set of warm objects to be those objects with a temperature between 21 and 26 degrees Celsius, we can see immediately that something with a temperature of 25 degrees Celsius is warmer than an object at 22 degrees Celsius. There are fractional degrees of membership between zero and one. Likewise, there are degrees of non-membership. Objects with temperatures of 14 and 20 degrees Celsius have different degrees of non-membership of the set of warm objects as defined above.

Again simplistically Intuitionistic logics are systems of *symbolic logic* that differ from *classical logic* by more closely mirroring the notion of *constructive proof*. In particular, systems of intuitionistic logic do not always include the *law of the excluded middle* and *double negation elimination*, which are fundamental inference rules in classical logic. Intuitionistic fuzzy logic (IFL) combines aspects of fuzzy logic and intuitionistic logic in a formal sense.[4]

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[3] Nishad, T.M., B. Mohaned Harit and A. Prasama. 2022. :Group Decision Making in Conditions of Uncertainty using Fermat's Weak Fuzzy Graphs and Beal's Weak Fuzzy Graphs." *Ratio Mathematica.* 41: 208-216.

[4] Atanassov, K.T. and A.G. Shannon, 1998. "A note on intuitionistic fuzzy logics," *Acta Philosophica*. 7 (1): 121-125.

IFL is actually a generalisation of both the mathematical intuitionism of Brouwer[5],[6] and the fuzzy sets of Zadeh[7].

Elements

Here we introduce the elements of an intuitionistic fuzzy propositional calculus (IFPC), basing our constructions on the definition of the Intuitionistic Fuzzy Sets[8],[9]and using notation from the theory of propositional calculus[10].

To each proposition (in the classical sense) one can assign its truth value: truth - denoted by 1, or falsity - 0. In the case of fuzzy logics this truth value is a real number in the interval [0, 1] and can be called the "truth degree" of a particular proposition. Here we add one more value - "falsity degree" - which will be in the interval [0, 1] as well. Thus, one assigns to the proposition *p* two real numbers $\mu(p)$ and $\gamma(p)$ in interval [0, 1] with the following constraint to hold:

$$\mu(\mathrm{p}) + \gamma(\mathrm{p}) \leq 1 .$$

Let this assignment be provided by an evaluation function *V* defined over a set of propositions *S* in such a way that

$$V(p) = \langle \mu(p), \gamma(p) \rangle .$$

Hence, the function $V : S \rightarrow [0,1] \times [0,1]$ gives the truth and falsity degrees of all propositions in *S*. We also assume that the evaluation function *V* assigns to the logical truth $T : V(T) = \langle 1,0 \rangle$, and to $F : V(F) = \langle 0,1 \rangle$. We shall discuss below the truth and falsity degrees of propositions which result from the application of logical operations (unary and binary) over input propositions which have known values according to a given evaluation function.

The evaluation of the first (classical) negation $\neg p$ of the proposition *p* can be defined through

$$V(\neg p) = \langle \gamma(p), \mu(p) \rangle .$$

When $\gamma(p) = 1 - \mu(p)$, i.e.,

[5] Dummett, M. 1977. *Elements of Intuitionism*. Oxford: Clarendon Press.
[6] Brouwer, L.E.J. 1975. *Collected Works* (edited by A. Heyting). Amsterdam: North Holland.
[7] Zadeh, L. 1965. "Fuzzy sets." *Information and Control*. 8: 338-353.
[8] Atanassov, Krassimir T. 2017. *Intuitionist Fuzzy Logics.* Cham, SUI: Springer.
[9] Atanassov, K. and G. Gargov. 1998. "Elements of Intuitionistic Fuzzy Logic. I." *Fuzzy Sets and Systems.* 95 (1): 39-52.
[10] Mendelson, E. 1964. *Introduction to Mathematical Logic.* Princeton, NJ: Van Nostrand.

$$V(p) = \langle \mu(p), 1 - \mu(p) \rangle,$$

and for $\neg p$ we get

$$V(\neg p) = \langle 1 - \mu(p), \mu(p) \rangle,$$

which coincides with the result for ordinary fuzzy logic[11]. Now, in IFL there are 53 other definitions of intuitionistic fuzzy negations[5].

When the values *V*(*p*) and *V*(*q*) of the propositions *p* and *q* are known, the evaluation function *V* can also be extended for the operations "&", "*v*" through the definitions

$$V(p \,\&\, q) = \langle \min(\mu(p), \mu(q)), \max(\gamma(p), \gamma(q)) \rangle,$$

and

$$V(p \underline{\vee} q) = \langle \max(\mu(p), \mu(q)), \min(\gamma(p), \gamma(q)) \rangle.$$

Depending on the way the operation "$\supset$" is defined, different variants of IFPC can be obtained. The form of an intuitionistic fuzzy implication is V(p ⊃ q) = <max(μ(q), η(p)), min(μ(p), η(q))>,

$$V(p \supset q) = \langle 1 - (1 - \mu(q)) \cdot sg(\mu(p) - \mu(q)), \gamma(q) \cdot sg(\mu(p) - \mu(q)) \cdot sg(\gamma(q) - \gamma(p)) \rangle,$$

in which

$$sg(x) = \begin{cases} 1 & if\ x > 0, \\ 0 & if\ x \leq 0. \end{cases}$$

When $p, q \in \{F, T\}$, the function *V* has the values displayed in Table 1for both implications. In[5] 190 different intuitionistic fuzzy implications are given. Each intuitionistic fuzzy implication generates 3 conjunctions and 3 disjunctions.

Table 1: Example of *V*(*p*⊃*q*

p	*V*(*p*)	*q*	*V*(*q*)	*V*(*p*⊃*q*)
F	**<0,1>**	*F*	**<0,1>**	**<1,0>**
F	**<0,1>**	*T*	**<1,0>**	**<1,0>**
T	**<1,0>**	*F*	**<0,1>**	**<0,1>**
T	**<1,0>**	*T*	**<1,0>**	**<1,0>**

[11] Negoita, C. and D. Ralescu. 1975. *Application of Fuzzy Sets to Systems Analysis.* Basel, SUI: Birkhäuser.

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By analogy with the operations over intuitionistic fuzzy sets (IFS), it will be convenient to define for the propositions $p, q \in S$ and for every intuitionistic fuzzy negation, conjunction, disjunction and implication

$$\neg V(p) = V(\neg p),$$

$$V(p) \wedge V(q) = V(p \,\&\, q),$$

$$V(p) \vee V(q) = V(p \underline{\vee} q),$$

$$V(p) \to V(q) = V(p \supset q).$$

For a given propositional form *A*:

- each proposition is a propositional form;
- if *A* is a propositional form, then $\neg A$ is a propositional form;
- if *A* and *B* are propositional forms, then $A \,\&\, B, A \underline{\vee} B, A \supset B$ are also propositional forms;
- *A* will be called a tautology if $V(A) =< 1,0 >$ for all valuation functions *V*.

Suppose that for the propositional forms *A* and *B*:

$$V(A) \le V(B) \text{ if, and only if, } (\mu_A \le \mu_B) \wedge (v_A \le v_B),$$

$$V(A) > V(B) \text{ if, and only if, } (\mu_A > \mu_B) \wedge (v_A < v_B).$$

More theory, relevant to the theme of this paper, particularly in relation to the development of the critical faculty in general education[12], can be explored by the interested reader.[13] IFL has a life of its own, apart from occasional use in the application of GNs[14].

For those more familiar with propositional calculus, the notion of an intuitionistic fuzzy set is a particular case of bilattice-based logics, wherein membership is evaluated by pairs of elements of the lattice $L_0 = \langle [0,1], \min, \max \rangle$.[15]

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[12]Shannon, A.G. 2018. "Intuitionistic Fuzzy Logic and Provisional Acceptance of Scientific Theories." In P. Angelov and S. Sotirov (eds). *Imprecision and Uncertainty in Information Representation and Processing. Studies in Fuzziness and Soft Computing 332*. Berlin: Springer, Ch.2, pp.15-23.

[13] Shannon, A.G. and K.T. Atanassov. 2004. "On Intuitionistic Fuzzy Multigraphs and Their Index Matrix Interpretations." In R. Yager and Vassil S. Sgurev (eds). *Intelligent Systems*. Sofia, Bulgaria and Piscataway, NJ: Institute of Electrical and Electronic Engineers, pp.440-443.

[14] Atanassov, K. 2007. *On Generalized Nets Theory*. Sofia: Prof. M. Drinov Academic Publishing House.

[15] Fitting, Melvin. 1989. "Bilattices and the Theory of Truth." *Journal of Philosophical Logic.* 18 (3): 225-256.

Concluding Comments

Try some of the ideas on organizational problems within your own department in the university, no matter how simplistic your initial effort might seem; for example,

Figure 1: Four quadrants of decision-making

IMPORTANT BUT NOT URGENT **← Degrees of Non-urgency** **↑ Degrees if Importance**	**IMPORTANT AND URGENT** **Degrees of Urgency →** **Degrees of Importance ↑**
← Degrees of Non-urgency **↓ Degrees of Unimportance** **NOT IMPORTANT AND NOT URGENT**	**Degrees of Urgency →** **Degrees of Unimportance ↓** **NOT IMPORTANT BUT URGENT**

Table 2: Selling or leasing a house at a time of high inflationary effects on mortgages

Options →	Live in	Lease	Sell
Advantages	Importance: capital gain	Mortgage pays rent	No mortgage
Disadvantages	Urgency: Inflation rate	High capital gains tax	Lost asset

Sim, Kwang Mong. 2001. "Bilattices and Reasoning in Artificial Intelligence: Concepts and Foundations." *Artificial Intelligence Review.* 15 (3): 219-240.